Rogue waves and rational solutions of the Boussinesq and nonlinear Schrödinger equations

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Abstract:

In this talk, I shall discuss special polynomials associated with rational solutions of the Painlevé equations and of the soliton equations solvable by the inverse scattering method, specifically for the Boussinesq and nonlinear Schrödinger equations.

The Painlevé equations are six nonlinear ordinary differential equations that have been the subject of much interest in the past thirty years, and have arisen in a variety of physical applications. Further the Painlevé equations may be thought of as nonlinear special functions. Rational solutions of the Painlevé equations are expressible in terms of the logarithmic derivative of certain special polynomials. For the fourth Painlevé equation these polynomials are known as the *generalized Hermite polynomials* and *generalized Okamoto polynomials*, derived by Noumi and Yamada [1]. The locations of the roots of these polynomials have a highly symmetric (and intriguing) structure in the complex plane [2].

It is well known that soliton equations have symmetry reductions which reduce them to the Painlevé equations. For example, scaling reductions of the Boussinesq and nonlinear Schrödinger equations are expressible in terms of the fourth Painlevé equation. Hence rational solutions of these equations can be expressed in terms of the generalized Hermite and generalized Okamoto polynomials. Further there are generalizations of these rational solutions of the Boussinesq and de-focusing nonlinear Schrödinger equations, which involve arbitrary parameters [3,4].

I shall also describe some additional rational solutions of the Boussinesq and focusing nonlinear Schrödinger equations which have applications to rogue waves [5].

References:

1. M Noumi and Y Yamada, *Nagoya Math. J.*, **153** (1999) 53–86.
2. P A Clarkson, *J. Math. Phys.*, **44** (2003) 5350–5374.
3. P A Clarkson, *Europ. J. Appl. Math.*, **17** (2006) 293–322.
4. P A Clarkson, *Anal. Appl.*, **6** (2008) 349–369.
5. A Ankiewicz, P A Clarkson and N Akhmediev, "*Rogue waves, rational solutions, the patterns of their zeros and integral relations*", preprint (2009).